# Research on Design Model Optimization of Mooring System in Near-Shallow Sea Observation Network

**Yuhua Su[1, a], Wei Jiang[2,b]**

[1]School of Mathematics and Computer Science, Hezhou University, Hezhou, Guangxi, 542899, China

[2]Department of Personnel, Hezhou University, Hezhou, Guangxi, 542899, China

[a]107913494@qq.com, [b] 122484412@qq.com

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**Abstract:** This paper uses the relevant data provided by the 2016 National Undergraduate Mathematical Modeling Contest and establishes relevant models to optimize the mooring system under specific sea conditions. Firstly, the force analysis of each part of the mooring system and the mooring system is carried out to obtain the force balance equation. Secondly, the model is established by taking the draft depth of the buoy, the maximum swimming radius and the inclination of the steel drum as the objective function. Iteratively solves the coordinates of each node, the inclination angle of the steel barrel, the angle between the chain and the seabed, and obtains the optimal state that the mooring system can operate well. Finally, the model is verified by Mathematica software and the anchor chain is traced.

## 1. Introduction

The transmission node of the near-shallow sea monitoring network is composed of a hydroacoustic communication system, a buoy system and a mooring system. The mooring system is an important source of information data of the marine monitoring information network. How to design the optimal mooring system to promote the signal transmission is of great significance. In this paper, the relevant data provided by the 2016 National College Students Mathematical Modeling Contest A is used to establish the relevant model to optimize the mooring system under certain sea conditions, so that the hanging heavy ball makes the steel barrel as vertical as possible (to tilt the steel drum) The angle needs to be controlled within 5°. In order to ensure that the mooring system is not blown away by the sea breeze, the angle between the anchor point and the seabed is required to be less than 16°.

## 2. Model assumptions

(1) It is assumed that the buoyancy of steel pipes, heavy balls and anchor chains in the mooring system is negligible.

(2) Assume that the local gravity acceleration is 9.8m/s^2 during the solution.

(3) It is assumed that the friction between objects in the mooring system is negligible.

(4) It is assumed that the center of gravity of the buoy is at the bottom at this time, and the buoy does not tilt during the swimming process.

(5) It is assumed that both the wind speed and the current velocity are along the horizontal direction.

## 3. Model establishment

(1) Force analysis of buoys

The force analysis of the buoy is shown in Figure 1. For the sake of simplicity, each physical quantity in the model appears.

The International System of Units is used to represent and omit its units.

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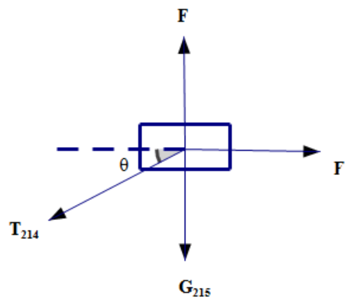


Figure 1 Schematic diagram of the force analysis of the buoy

From the above force analysis, the buoyancy of the seawater obtained by the buoy can be obtained as:

$$F_b = \rho_{water} g V_{away} = 1.025 \times 10^3 \times 9.8 \times \pi \left(\frac{d}{2}\right)^2 \hbar \quad (1)$$

The buoy diameter is known to be d=2.

The wind on the buoy:

$$F_{wind} = 0.625 S v^2 = 0.625 \cdot d \times (2 - \hbar) v^2$$

Buoy's gravity:

$$G_{buoy} = M_b g = 1000 \times 9.8 = 9\,800$$

Where M_b denotes the mass of the buoy; S denotes the projected area of the object on the wind direction plane; v denotes the sea surface wind speed; V_away denotes the volume of the buoy immersed in the seawater.

Note that the tensile force of the steel pipe on the buoy in Section 4 is T_215, and the external force is zero:

$$T_{215} \cos \theta_{215} = F_{wind}$$

$$T_{215} \sin \theta_{215} + G_{buoy} = F_b$$

(2) For each section of the steel pipe to be subjected to force analysis, the following is to select the steel pipe 215 for force analysis, as shown in Figure 2:

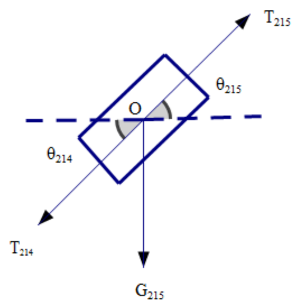


Figure 2 Schematic diagram of force analysis of steel pipe

$$G_{215} = M_{Steel\ Pipe} \cdot g = 98$$

$$\begin{cases} T_{215} \cdot \cos \theta_{215} = T_{214} \cdot \cos \theta_{214} \\ T_{215} \cdot \sin \theta_{215} = T_{214} \cdot \sin \theta_{214} + G_{215} \end{cases}$$

Where, G_215 indicates the gravity of the steel pipe 215, T_214 indicates the tensile force generated by the steel pipe 214 on the steel pipe 215, and θ_214 indicates the angle formed by the tensile force T_214 and the horizontal direction.

The remaining steel pipes can be similarly derived:

$$\begin{cases} T_i \cos\theta_i = T_{i-1} \cos\theta_{i-1} \\ T_i \sin\theta_i = T_{i-1} \sin\theta_{i-1} + G_i \end{cases}$$

Where T_i represents the tensile force of the steel pipe i to the steel pipe i-1, θ_i represents the angle formed by T_i and the horizontal direction, and G_i represents the gravity of the steel pipe i (i = 214, 213, 212).

(3) Perform the force analysis on the steel drum and the heavy ball as a whole, as shown in the following figure.

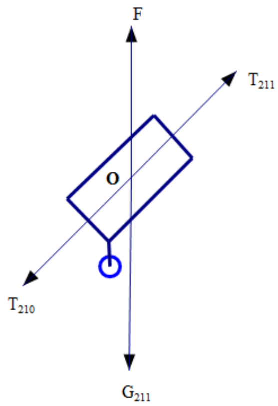


Figure 3 Schematic diagram of the overall analysis of steel drums and heavy balls

Steel drum gravity: $G_{Steel\ drum} = M_{Steel\ drum} \cdot g = 980$

Heavy ball gravity: $G_{Heavy\ ball} = M_{Heavy\ ball} \cdot g = 1\,200 \times 9.8 = 11\,760$

The buoyancy of the steel drum:

$$F_{Steel\ drum\ buoyancy} = \rho g V_{Steel\ drum} = 1.025 \times 10^3 \times 9.8 \times \pi \left(\frac{0.3}{2}\right)^2 \times 1$$

$$G_{211} = G_{Steel\ drum\ buoyancy} + G_{Heavy\ ball} - F_{Steel\ drum\ buoyancy}$$

$$= 98 + 1\,200 \times 9.8 - 1.025 \times 10^3 \times 9.8 \times \pi \left(\frac{0.3}{2}\right)^2 \times 1$$

When the force is balanced, it is satisfied:

$$\begin{cases} T_{211} \cos\theta_{211} = T_{210} \cos\theta_{210} \\ T_{211} \sin\theta_{211} = T_{210} \sin\theta_{210} + G_{211} \end{cases}$$

(4) For the analysis of the 210-section anchor chain one by one, it is only necessary to analyze the i-th chain anchor chain Pi−1Pi, as shown in the following figure.

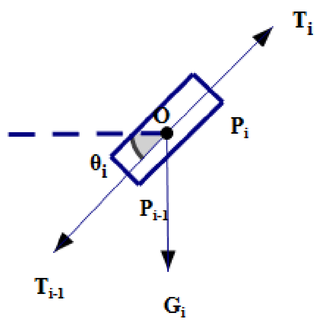


Figure 4 Schematic diagram of the force analysis of the i-th anchor chain

The gravity of each section of the anchor chain is:

$$G_i = 0.105 \times 7 \times 9.8 = 7.203$$

$$\begin{cases} T_i\cos\theta_i = T_{i-1}\cos\theta_{i-1} \\ T_i\sin\theta_i = T_{i-1}\sin\theta_{i-1} + G_i \end{cases}$$

(5) The mooring system (including anchor chain, steel drum, heavy ball, steel pipe, buoy) is analyzed as a whole, as shown in Figure 6.

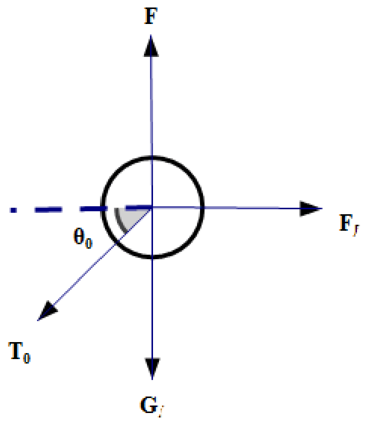


Figure 5 Overall force analysis of mooring system

Total gravity of the system

$$= G_{Anchor\ chain} + G_{Steel\ drum} + G_{Heavy\ ball} + G_{Steel\ Pipe} + G_{buoy}$$

$$= (22.05 \times 7 + 100 + 1\,200 + 10 \times 4 + 1\,000) \times 9.8$$

At this time, the buoyancy of the mooring system is:

$$F_b = \rho g V_w = 1.025 \times 10^3 \times 9.8 \times \pi[\left(\frac{d}{2}\right)^2 \times h + 0.15^2]$$

The tension of the anchor to the first section of the anchor chain is denoted as T_0, and the following equilibrium equation can be listed:

$$\begin{cases} T_0\cos\theta_0 = F_{wind} \\ T_0\sin\theta_0 = F_b - G_t \end{cases}$$

Where θ_0 represents the angle formed by T_0 and the horizontal direction.

By combining the gravity received by the steel drum and the weight ball with the buoyancy of the steel drum, it can be seen that the force of all the remaining objects of the segment 211 and the segment 1-215 is consistent. The force balance equation of the object 211 and the balance equation of the object 1-215 can be uniformly expressed:

$$\begin{cases} T_0\cos\theta_0 = F_{wind} \\ T_0\sin\theta_0 = F_b - G_t \end{cases}$$

$$\begin{cases} T_i\cos\theta_i = T_{i-1}\cos\theta_{i-1} \\ T_i\sin\theta_i = T_{i-1}\sin\theta_{i-1} + G_i \end{cases}$$

## 4. Model solution

Study the sequence $\{T_i\cos\theta_i\}_{i=1}^{215}$, in the formula (20), which is recursively known to be a constant column, ie

$$T_i\cos\theta_i \equiv F_{wind} \qquad i = 1,2,\ldots\ldots,\ 215$$

The general term of the series $\{T_i\sin\theta_i\}$ is expressed as follows:

$$T_i \sin\theta_i = T_0 \sin\theta_0 + \sum_{k=1}^{i} G_k = F_b - G_t + \sum_{k=1}^{i} G_k$$

Then the general formula of the series $\{T_i \cos\theta_i\}_{i=1}^{215}$ and $\{T_i \sin\theta_i\}_{i=1}^{215}$ is obtained:

$$\begin{cases} T_i \cos\theta_i = F_{wind} \\ T_i \sin\theta_i = F_b - G_t + \sum_{k=1}^{i} G_k \end{cases}$$

$$T_i = T_i(\hbar) = \sqrt{F_{wind}^2 + \left(F_b - G_t + \sum_{k=1}^{i} G_k\right)^2}$$

$$\cos\theta_i = \frac{F_{wind}}{T_i} = \frac{F_{wind}}{\sqrt{F_{wind}^2 + \left(F_b - G_t + \sum_{k=1}^{i} G_k\right)^2}}$$

$$\sin\theta_i = \frac{F_b - G_t + \sum_{k=1}^{i} G_k}{T_i} = \frac{F_b - G_t + \sum_{k=1}^{i} G_k}{\sqrt{F_{wind}^2 + \left(F_b - G_t + \sum_{k=1}^{i} G_k\right)^2}}$$

Let the length of the $P_{i-1}P_i$ segment be $L_i$, then

$$L_1 = L_2 = \cdots \ldots = L_{210} = 0.105$$

$$L_{211} = L_{212} = L_{213} = L_{214} = L_{215} = 1$$

Let $P_i(x_i, y_i)$, the coordinates of the node P_i can be calculated as follows:

$$x_1 = L_1 \cos\theta_0$$

$$x_2 = x_1 + L_2 \cos\theta_1$$

$$x_i = \sum_{k=1}^{i} L_k \cos\theta_{k-1}$$

$$y_i = \sum_{k=1}^{i} L_k \sin\theta_{k-1}$$

$$y_{215} = 18 - \mathrm{h}$$

In order to understand the equation y215=18−h, layer-by-layer substitution is required, and the calculation amount is large. This paper establishes the following optimized nonlinear programming model:

$$\min\ Z = |y_{215} - (18 - \hbar)|$$

$$0 \le \hbar \le 2$$

Using Lingo9 software to solve h≈0.742 866 8 . The tensile force T_i and the coordinate P_i (x_i, y_i) of each segment $P_{i-1}P_i$ can be obtained from h. Some of the calculation results are as follows:

When v=36m/s, calculate the coordinates $P_i(x_i, y_i)$ of each node according to the model (i= 1, 2, ..., 215), draw a scatter plot, and get the shape of the anchor chain as shown below:

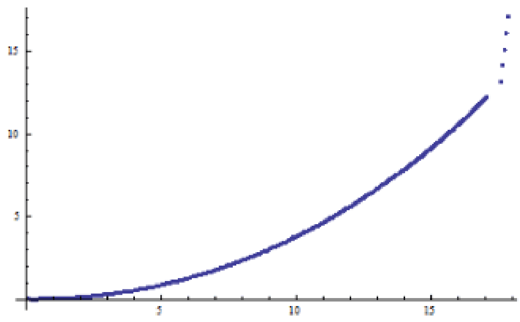


Figure 6 w=36m/s, m=1200kg, the shape of the anchor chain

By solving the calculation, the radius of the circle of the buoy swimming $\mathrm{r} = x_{215} = 19.2525$ is obtained, and the inclination angle of the steel drum is 7.93445 degrees. At this time, the longitudinal inclination angle of the steel drum has exceeded 5°, and the anchor chain and the seabed The angle solution $\theta_0$=17.9691 $^{\circ}$ has also exceeded 16°. Obviously, the result of the model solution does not match the actual requirements, and the anchor chain may be towed away. In order to make the equipment work normally, At this time, it is necessary to adjust the mass m of the weight ball.

**5. Establishing a planning model to realize the optimal design of the mooring system**

In order to properly select the mass m of the weight ball, so that the steel drum is as far as possible, so that the angle between the anchor chain and the seabed becomes smaller, the following model is established:

$$minz = sin\left(\frac{\pi}{2} - \theta_{211}\right) + sin\,\theta_0$$

$$y_{215} = 18 - \hbar$$

In addition, it is necessary to ensure that the longitudinal inclination of the steel pipe does not exceed$5^{\circ}$($\theta_{211} \geq 85^{\circ}$)to ensure that the equipment in the steel drum works well:

$$cos\,\theta_{211} \leq cos\,85^{\circ}$$

At the same time, in order to prevent the anchor from being blown away by the sea breeze, the angle between the first section of the anchor chain and the seabed cannot exceed$16^{\circ}$ ($\theta_0 \leq 16^{\circ}$), then

$$cos\,\theta_0 \geq cos\,16^{\circ}$$

In addition, m can not be too large, otherwise the buoy will sink into the sea. When the buoy has just been immersed in seawater, a force analysis of the mooring system can be obtained:

$$F_{Immersion\ float} = \rho g V = 1.025 \times 10^3 \times 9.8 \times \pi \times \left(\frac{2}{2}\right)^2 \times 2$$

$$G_t = (22.05 \times 7 + 100 + m + 10 \times 4 + 1\,000) \times 9.8$$

$$F_b = T_0\,sin\,\theta_0 + G_t \geq G_t$$

$$1.025 \times 10^3 \times 9.8 \times \pi \times \left(\frac{2}{2}\right)^2 \times 2 \geq (22.05 \times 7 + 100 + m + 10 \times 4 + 1000) \times 9.8$$

$$m \leq 5\,142.65$$

$$m > 1\,200$$

At this point, the planning model has been established:

$$\min\ z = sin\left(\frac{\pi}{2} - \theta_{211}\right) + sin\,\theta_0$$

$$\begin{cases} y_{215} = 18 - h \\ cos\,\theta_{211} \le cos\,85^{\circ} \\ cos\,\theta_0 \ge cos\,16^{\circ} \\ 0 < h \le 2 \\ 1\,200 < m \le 5\,142.65 \end{cases}$$

The Lingo code for solving the model can be found in the appendix, and m=1757.990, h=0.940 0013 can be obtained. By returning m and h, the data of each point coordinate can be obtained. According to the point coordinates $P_i(x_i, y_i)$ ( $i = 1,2, \ldots\ldots, 215$ ) , the shape of the anchor chain can be obtained by drawing the scatter plot as shown below:

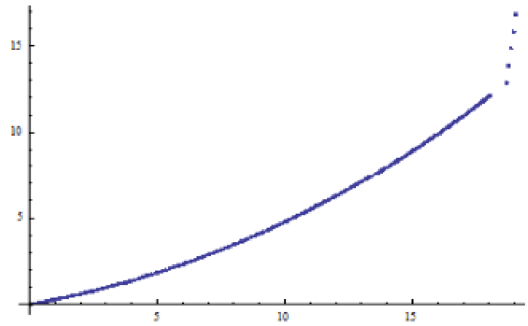

Figure 7 Anchor chain shape after hanging a heavy ball

Using Mathematica software to solve the problem, the radius of the floating circle of the buoy is $r = x_{215} = 19.061\,7$. The angle between the anchor and the seabed is 14.5547°, and the longitudinal inclination of the steel barrel is 5.00058°, where the inclination angles of the four steel pipes are respectively 85.024°, 85.0495°, 85.0742°, 85.0986°; due to the software precision design problem, and the seawater pair is not considered in this paper. The buoyancy of the anchor chain, steel pipe and heavy ball makes the result a certain error. At this time, the inclination angle of the steel drum 5.00058° can be approximated as 5°, which meets the requirements of the normal operation of the mooring system. The working equipment can be operated well.

## 6. Conclusion

### 6.1 The Advantages of the Model

(1) The relevant mathematical models established in this paper are closely related to practical problems, thus making them clear and easy to understand.

(2) In this paper, the force analysis of each part and the whole of the mooring system is carried out. The model is established according to the equilibrium condition, and the calculation is simplified by solving the recursive relation. The modeling method is simple and easy to popularize.

(3) According to the practical application, there are also restrictions on the design of mooring system. This paper establishes a planning model, and converts those qualifications into equality constraints and inequality constraints, so that the solved results can meet the actual results. Demand, the planning model is relatively simple and practical, and the application is also extensive.

### 6.2 The Disadvantages of the Model

(1) For the convenience of the analysis process, this paper does not consider the buoyancy of seawater on the anchor chain, steel pipe and heavy ball, so that the result will have a certain error.

(2) For the sake of simplicity, it is assumed that the center of gravity of the buoy is at the bottom, and the tilt of the buoy is not considered, which is inconsistent with the reality.

(3) Because the solution of this model is relatively complicated, the solution obtained by Lingo

software may be only the local optimal solution and it is difficult to find multiple solutions or the overall optimal solution.

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